FACTOR SHARES, ECONOMIC GROWTH, AND THE INDUSTRIAL REVOLUTION

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Despite the transformative nature of the period, growth in the total factor productivity (TFP) residual during the British industrial revolution is typically estimated to have been relatively slow. There is no theoretical reason, however, to limit the manifestation of technical progress to changes in the TFP parameter. This paper acknowledges recent models of share-altering technical progress and provides a growth accounting methodology that allows for variable factor shares. Virtually all existing growth accounting studies pertaining to the British industrial revolution assume that factor shares were constant parameters. The data, however, indicate that factor shares were not constant; labor and land shares decreased during the period while physical capital's share increased. Implementing a variable factor share methodology reveals that productivity growth rates during the industrial revolution, though modest, were faster than previously thought. Moreover, the variation in factor shares is found to explain up to 4.7% of the variation in output per worker growth, which represents nearly 1/5 of the explanatory power accruing to all observables over the period 1760-1860.

Introduction

The current consensus among economic historians is that productivity growth was relatively slow during the British industrial revolution. This consensus is primarily predicated on growth accounting analyses that virtually always assume constant factor shares. This paper relaxes that assumption. We acknowledge the variation in factor shares exhibited by the data, present a growth accounting methodology that is capable of handling variable factor shares, and provide results that i) indicate productivity growth was faster than is currently thought and ii) reveal a non-trivial role for factor share variation in explaining output per worker growth.

When we relax the standard assumption that physical capital's share is constant and equal to 0.35, we find that the absolute value of the capital per worker growth rate is reduced in each of the ten decades between 1760 and 1860. The reduction is largest for the period 1800-1810, when the growth rate falls by about 2.5 percentage points. In addition, allowing for variation in land's share, as opposed to assuming it is constant and equal to 0.15, has a non-trivial impact on land per worker growth rates. The biggest impacts are for 1790-1800 and 1840-1850, when the absolute values of land per worker growth rates increase and decrease by about 1 percentage point, respectively.

The most substantial impact on the growth accounting results is seen via the change in the growth rate of the productivity residual. Relative to the standard constant share specification, growth in the productivity residual increases in seven out of ten decades when factor shares are allowed to vary over time. In two of these decades, 1800-1810 and 1840-1850, growth in the productivity residual changes from negative to positive.

Intertemporal variation in factor shares, in addition to altering input and productivity growth rates, has explanatory power in and of itself. We find that variation in factor shares explains as much as 4.7% of the variation in output per worker growth over the period 1760-1860, which accounts for nearly 1/5 of the explanatory power accruing to all observables.

The remainder of the paper is organized as follows. First, we discuss the industrial revolution and factor shares. Second, we discuss the unit-invariance problem that arises when time-varying factor shares are incorporated into the standard growth accounting framework and present a new framework that circumvents the problem. Third, we describe the data. Fourth, we present our growth accounting results. Fifth, we present and use a variance decomposition methodology to determine the explanatory power of factor shares. We close with some brief concluding remarks.

Background on the Industrial Revolution

The idea of "the Industrial Revolution as a single great historical event" was popularized by Arnold Toynbee in a series of lectures he gave in the 1880s.[1] Although Toynbee's focus was at least as much on the social effects of industrialization as on industrialization itself, the idea of an "Industrial Revolution," capital I, capital R, was established. In the decades prior to World War II, however, scholars backed off of this interpretation focusing more on gradual rather than revolutionary economic changes during this period.[2]

The idea of the industrial revolution as indeed revolutionary was revived in the post-war period by T.S. Ashton and his (in)famous schoolboy's "not inapt… answer to a question on the industrial revolution" which began: "About 1760 a wave of gadgets swept over England" (1948, 58). The list of new inventions and innovations from this period is indeed impressive. Hargreaves' spinning jenny (1764), Arkwright's water frame and Watt's separate condenser improvement on the steam engine (1769),

[1] The quote is from Toynbee's nephew, Arnold J. Toynbee's, preface to the 1956 reprint of Toynbee's *Lectures on the Industrial Revolution in England* which was originally published posthumously in 1884.

[2] Emma Griffin's "The 'industrial revolution': interpretations from 1830 to the present" provides a nice overview of the changing estimates of economic output and growth. Joel Mokyr's (1993) "Editor's Introduction" is perhaps the best short introduction to the Industrial Revolution in general.

Crompton's spinning mule (1774), Cartwright's power loom (1785), and Cort's "puddling and rolling" process for iron (1785) all represented significant technological advances. Ashton argued that these, and other innovations across the economy, "surged up with a suddenness for which it is difficult to find a parallel at any other time or place… and can hardly be told in terms of an evolutionary process" (1948, 58).

In the 1960s, the view of the industrial revolution as a period of truly extraordinary economic change reached its zenith. Walt Rostow's (1960) influential *The Stages of Economic Growth* argued that the British economy underwent a discrete "take-off" period in the late eighteenth century. Other authors agreed, including Ronald Hartwell who wrote, "on any historical accounting, the industrial revolution is one of the great discontinuities of history; it would not be implausible indeed, to claim that it has been the greatest" (1965, 165). David Landes' (1969 and 1993) assessment was somewhat tempered but still argued in favor of the significant transformative interpretation, "The Revolution was a revolution. If it was slower than some people would like, it was fast by comparison with the traditional pace of economic change" (1993, 170).

Empirical work by Walter Hoffmann (1955) and especially that of Phyllis Deane and W.A. Cole in the 1960s found that while economic growth during the industrial revolution was perhaps not as profound as Rostow and Hartwell suggested, it was still significant. Deane and Cole (1967) estimated that annual growth in British economic output was slightly more than three percent annually in the first three decades of the nineteenth century, with corresponding per-capita annual growth rates of 1.5 percent.

In the 1980s, research by Nicholas Crafts and Knick Harley led to the modern view of the industrial revolution as not being very revolutionary at all, at least in terms of immediate significant increases in productivity and economic growth. Harley (1982) argued that the growth estimates of Hoffmann and Deane and Cole were as much as fifty percent too high because the former over-emphasized the cotton industry while the latter had done the same for foreign trade. Similarly, Crafts (1980) argues that Deane and Cole's estimates overstated real economic growth in the early nineteenth century due to the use of incorrect deflators for national output.

Crafts (1983) does suggest, though, that his estimates are basically just a refinement of Deane and Cole's and that his basic findings are much more similar to those of Deane and Cole than they are to estimates of previous economic historians. Crafts (1985) computed economic growth rates to be about one third lower than Deane and Cole's estimates for 1780-1831. His estimates for annual growth in total factor productivity (TFP) in industry were 0.2 percent from 1760-1801 and 0.4 percent from 1801-1831, while his estimates for annual TFP growth for the overall economy were 0.2 percent and 0.7 percent for the same years.

The Crafts-Harley view of mild economic growth with limited growth in productivity has not gone uncontested. Maxine Berg and Pat Hudson (1992) argue that Crafts and Harley's estimates of growth in output and productivity are too low and then further criticize the use of TFP to measure the impact of the industrial revolution as TFP measures are too limited to capture the full scope of the changes taking place during the period. David Greasley and Les Oxley (1994) applied unit root tests to Craft and Harley's estimates of industrial production from 1780-1851 and found that the estimates were non-stationary and thus concluded that the industrial revolution did indeed mark a historical discontinuity. In response to their critics, Crafts and Harley (1992) presented revised estimates of economic growth and growth in productivity. These new estimates suggested that productivity growth had been even lower than what they had previously found averaging only 0.1 percent annually from 1760-1801 and 0.35 percent annually from 1801-1831. Crafts and Harley specifically do not reject the view that the industrial revolution was a period of unique and remarkable socio-economic structural change, but they argue against the idea that industrial innovations led to a rapid increase in economic growth.

How then are the estimates for low productivity growth reconciled with the technological achievements of the industrial revolution? The primary answer is that the industrial revolution was initially limited in the scope of industries it affected. In particular, the use of steam engines was not diffuse and therefore had only a relatively minor impact throughout the economy (Crafts 2004a, 2004b, 2008, 2014b). Although Peter Temin (1997) argues against this narrow view of productivity growth being limited to only a few industries, his view is not widely held at the present time. Gregory Clark (2001), for example, argues that the limited productivity growth that did occur during this period was mostly confined to textile manufacturing.

The study of output and productivity growth during the industrial revolution has remained an active area of research. Crafts (2014a), using new estimates of land and real GDP growth, re-estimated TFP growth and found that it was 0.4 percent per year for both the 1760-1800 and 1800-1830 periods. The estimate for the earlier period is double that of his previous estimate, while the estimate for the latter period is slightly below his previously estimated 0.5 percent. In both cases, the productivity growth rates are generally consistent with the original Crafts-Harley view. Using the alternative “dual technique” Pol Antras and Hans-Joachim Voth (2003) generated estimates of productivity growth from factor prices. They found very low rates of productivity growth prior to 1800 and only very modest productivity growth in the early decades of the nineteenth century. Antras and Voth suggest that while their results may not supersede those from previous studies, they provide additional support for the Crafts-Harley view of the industrial revolution.

Our current paper is similar to the Antras and Voth paper in that it also applies a new method of estimating productivity growth to the period of the industrial revolution. The following sections of this paper describe some of the features of previous research on the subject and explain how our work builds and improves upon those studies by relaxing one of the standard assumptions of most of this work: that factor shares were constant.

The Constant Factor Share Assumption

The assumption that factor shares were constant during the industrial revolution is typical in studies of the period. Crafts (1985) states that “physical capital’s share, labor’s share and land’s share are taken to be 0.35, 0.5 and 0.15 respectively.” Crafts gives no details about the derivation of these values but says the estimates are “very similar to Deirdre McCloskey’s (1981) figures.” McCloskey uses land rents to estimate land’s share for four years: 1760, 1800, 1830 and 1860. From these four estimates, McCloskey computes an arithmetic average of 0.13, and, despite the fact that her estimates range from 0.18 in 1760 to 0.085 in 1860, she assumes that 0.13 is typical of the entire period 1780-1860.

McCloskey uses the Deane and Cole (1967) estimates of wages and salaries in 1801, 1831, and 1861 to estimate labor's share. She assumes the 1760 value equals the 1801 value and computes labor's share as $wage\ and\ salaries / GDI$, where *GDI* is gross domestic income and equal to gross national income less income from abroad. The arithmetic average for her labor share estimate is 0.46. McCloskey computes capital's share as a residual and finds it to equal 1.0 - 0.46 - 0.13 = 0.41.

The shortcomings of McCloskey's estimates stemming from data constraints are obvious and even McCloskey admits that the factor share estimates are "crude." In addition to the data issues, which accompany most industrial revolution analyses, there are computational inconsistencies and omissions in McCloskey's approach that lead to further imprecision in her estimates. First, the denominators in her labor and land share estimates are not the same. McCloskey does not explicitly state what the denominator in her land share estimate represents. The intention may be for the denominator to equal *GDI*, as in the labor share estimate, but the actual numbers in the denominators of the two share measures do not match up. Thus, the capital share residual is hard to interpret, and it is not clear what type of restriction is being imposed when factor shares do not have equivalent denominators but are assumed to sum to 1. Second, McCloskey ignores indirect taxes and by doing so implicitly assumes that income received by firms but paid to the government in the form of taxes is generated by something other than factors of production. This approach skews the estimation. Specifically, factor share estimates computed directly will be understated, and those computed indirectly as a residual will be overstated.

An Updated Factor Share Data Set

Clark (2010) provides historical factor share estimates for Great Britain that are methodologically more appealing. He computes labor's share as $\frac{Wage\ Income}{NNI - Indirect\ Taxes}$ and land's share as $\frac{Land\ Rents}{NNI - Indirect\ Taxes}$ where *NNI* is net nominal national income. He then estimates physical capital's share as a residual. The denominators are the same for all shares; thus interpretation is straightforward. Ideally, indirect taxes should be allocated to labor, land or capital depending on the tax type, unfortunately the detailed data necessary for such a breakdown are unavailable. By subtracting indirect taxes, the implicit assumption is that the fractions of indirect taxes accruing to labor, land and capital are equivalent to labor, land and capital shares, respectively.

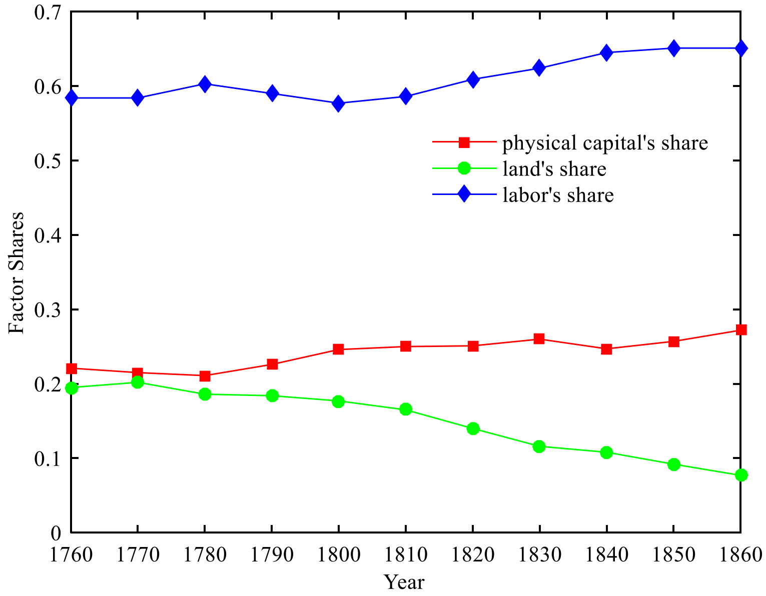
These shares are plotted against time in Figure 1. Notice that all values for physical capital's share fall below 0.35, the smallest of the standard parameter values inserted for physical capital's share in much of the Industrial Revolution growth accounting literature.[3] Over the period 1760-1860, physical capital's share is generally increasing and takes on its largest value (0.272) in 1860.[4] Land's share decreases over the period.

In contrast to the income-based approach that Clark uses to estimate GDP, Stephen Broadberry et al. use an output-based approach. We also used the Broadberry et al. data and compute alternative share estimates. In doing this, we still use Clark's wage income, land rent and tax data. Thus, these alternative shares only differ in the denominators so that labor and land shares are given by $\frac{\text{Wage Income}}{\text{GDP} - \text{Indirect Taxes}}$ and $\frac{\text{Land Rents}}{\text{GDP} - \text{Indirect Taxes}}$, respectively. These alternative shares are plotted as Figure A1 in Appendix 1. They are noisier than those reported by Clark, but the same general trends are revealed. We have a preference for the Clark methodology because his shares behave in a manner that is more in line with the

[3] Crafts (1985) and Antras and Voth (2003) both assume physical capital's share equals 0.35. Crafts (1995) and Feinstein (1981) assume physical capital shares of 0.4 and 0.5, respectively.

[4] Clark (2010) provides estimates of physical capital shares all the way back to 1200. Between 1200 and 1860, Clark never estimates a value for physical capital's share above 0.32. So, irrespective of the time period, 0.35 is too large a value for physical capital's share. Most of Clark's estimates are between 0.2 and 0.3, well below the standard parameter value used in the current literature.

economic theory discussed below. For this reason, we use Clark's factor shares for the analysis in the main text. We note that the results based on factor shares computed using Broadberry et al. estimates of GDP are not qualitatively different from those found when using Clark's factor shares. These additional results are provided as Table A1 in Appendix 1.



Source: Clark (2010).

Figure 1

Factor Shares Over Time (1760-1860)

Theoretical Support for Variable Factor Shares

There are two types of technological progress, factor eliminating and factor augmenting. Factor augmenting progress receives most of the attention in the economic growth literature. Such progress increases the number of "effective workers" or increases the number of "effective machines." This leads researchers to commonly associate technological progress with changes in the total factor productivity (TFP) parameter.[5] However, there is no theoretical reason to limit technological progress to changes in the aforementioned parameter (usually denoted as “*A*”) that enters the production function in a multiplicative manner.

Factor eliminating progress replaces raw labor and land with physical and human capital. Examples of progress of this kind occurred in the textile industry where mechanization, as noted by Harley (1998), greatly affected the production of fine yarn via the use of the water frame and spinning mule. Later the power-loom similarly transformed the weaving of cloth. Mining also experienced such progress as continued improvements in the efficiency of steam engines reduced the amount of coal needed to pump water from mines.

This type of progress manifests itself via changing factor shares. John Seater (2005), Hernando Zuleta (2008), and Pietro Peretto and Seater (2013) develop economic growth models where factor eliminating progress drives economic growth. Their models allow factor shares to change endogenously via spending on Research and Development (R&D). The R&D occurs to eliminate the non-reproducible factors of production (raw labor and land). As economies advance, non-reproducible factors are used less intensively, and reproducible factors (physical capital and human capital) are used more intensively.

Factor eliminating technological progress is not the only explanation for variable factor shares. Raw labor’s share may decline as a result of institutional changes that decrease the bargaining power of workers.[6] Zuleta (2012) discusses the possibility that aggregate factor shares can change if there is an increase in the relative size of production sectors

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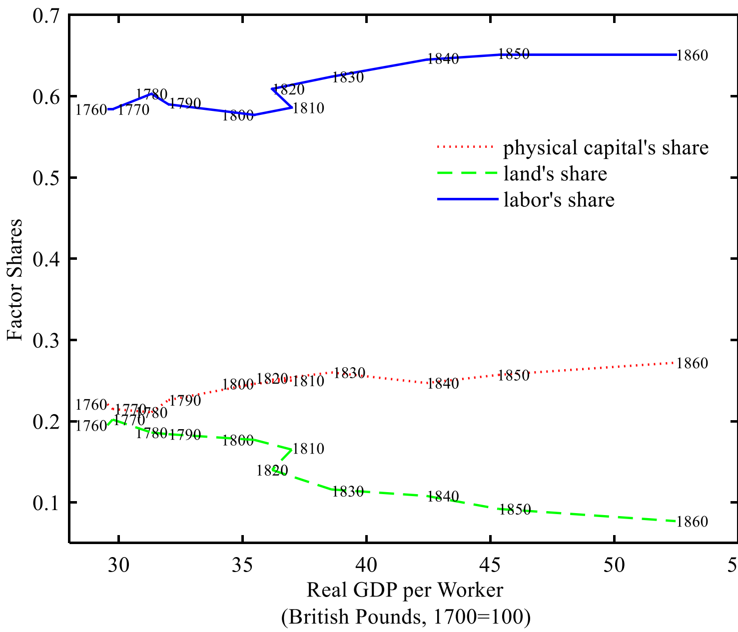
[5] Crafts (2004a) allows technological change to impact output per worker growth through embodied, capital deepening effects as well as through standard TFP residual growth. However, Crafts assumes factor shares are constant.

[6] See Zuleta (2012), Samuel Bentolila and Gilles Saint-Paul (2003), Nicola Giammarioli et al. (2002), Norbert Berthold et al. (2002) and Benjamin Bental and Dominique Demougin (2010).

where factor shares are different from the average. Zuleta (2007) explains that specialization according to comparative advantage can increase the share of abundant factors and decrease the share of scarce factors.

The variability in factor shares exhibited by Clark's data is consistent with the aforementioned explanations, particularly the models of factor eliminating progress. These models predict that reproducible and non-reproducible factor shares should increase and decrease, respectively, with output per worker. Notice in Figure 1 that physical capital's share and land's share start to diverge quite noticeably in 1780. Figure 2 shows that this divergence is accompanied by increases in output per worker.



Source: Clark (2010).

Figure 2

Factor Shares vs. Real GDP per Worker

Clark's labor share estimate is an amalgamation of the fractions of total income accruing to raw labor, a non-reproducible factor, and human capital, a reproducible factor. He does not separate the two factor shares, and theory makes no predictions about the intertemporal behavior of "total" labor's share. To our knowledge, other growth analyses of the industrial revolution do not separate raw labor from human capital either. This amalgamation is partially due to data limitations, but primarily due to the belief that human capital accumulation was a relatively unimportant aspect of the industrial revolution (Robert Allen 2009). Nonetheless, Figures 1 and 2 reveal that labor's share is not constant over time. It exhibits a slight positive trend and varies from 0.584 in 1760 to 0.651 in 1860. Thus, treating labor's share as a constant parameter is unwarranted.

A New Framework

Implementing a growth accounting methodology that allows for intertemporal variation in factor shares requires a departure from the standard Cobb-Douglas or Constant Elasticity of Substitution (CES) production function. If factor shares are allowed to vary over time, the results based on Cobb-Douglas or CES forms become sensitive to a simple change in units used to measure inputs. In other words, variable factor shares create a classic index number problem.

We perform the growth accounting exercise using a translog framework that is robust to the choice of measurement units in the presence of time-varying factor shares. To the best of our knowledge, ours is the first time the translog production function has been used to determine the relative importance of observables and unobservables in explaining economic growth during the industrial revolution.[7]

Because standard analyses treat factor shares as constant parameters, standard analyses attribute all variation in observable growth to variation

[7] Allen (2009) uses estimates of labor, land and capital shares for two years (1770 and 1860) to estimate the parameters of a translog production function, and then uses the estimated parameters to derive simulated values of macroeconomic aggregates. However, his focus is not on growth accounting but rather on explaining trends in real wages, productivity, profits and shares.

in the growth of factors of production.[8] When the constant factor share assumption is relaxed, variation in observable growth reflects variation in the growth of factors as well as variation in factor shares. Do the factor shares have explanatory power? Simply comparing the constant share estimate of the variance of output per worker growth explained by observable growth to its corresponding variable share estimate will not yield answers to such questions. It will only reveal how the combined explanatory power of all observable growth rates has changed. To determine the importance of growth in each individual factor and factor share in explaining intertemporal variation in output per worker growth, we separate the variation in output per worker growth explained by observable growth into that accruing to factors and that accruing to factor shares. The current literature makes no attempts to do this.

Growth Accounting with Variable Shares: The Unit-Invariance Issue

Let Y_t, K_t, L_t and N_t denote aggregate output, physical capital, raw labor and natural capital (land), respectively, at time *t*. Suppose the aggregate production technology is Cobb-Douglas so that the per worker production function is given by

$$y_t = A_t k_t^{\alpha_i} n_t^{\gamma_t} \tag{1}$$

where α_t is physical capital's share, γ_t is natural capital's share, A_t is productivity, *k* represents per worker physical capital, and *n* represents per worker natural capital. Notice that α and γ are indexed by *t* and allowed to vary over time. By taking natural logs and differentiating with respect to time, equation (1) can be expressed in growth rate form as

$$\frac{\dot{y}}{y} = \frac{\dot{A}}{A} + \alpha_t\left(\frac{\dot{k}}{k}\right) + \gamma_t\left(\frac{\dot{n}}{n}\right) + \left(\frac{\dot{\alpha}}{\alpha}\right)(\ln k_t)\alpha_t + \left(\frac{\dot{\gamma}}{\gamma}\right)(\ln n_t)\gamma_t \tag{2}$$

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[8]Antras and Voth (2003) do consider the sensitivity of their results to different factor share values, but they do not let shares vary over time. Shares are treated as constant parameters so the potential for explanatory power to accrue to factor shares is not present in their growth accounting analysis.

where a dot denotes the time derivative. With time-series data on y_t, k_t, n_t, α_t and γ_t, $\frac{\dot{A}}{A}$ can be computed as a residual.

Consider a change in the units used to measure *k*. Perhaps the original data is in dollars but is then multiplied by the constant $\theta = 1/1{,}000$ to convert the data into thousands of dollars. Now the fourth term on the right side of equation (2) is given by

$$\left(\frac{\dot{\alpha}}{\alpha}\right)[\ln(\theta k_t)]\alpha_t = \frac{\dot{\alpha}}{\alpha}[\ln\theta + \ln k_t]\alpha_t \tag{3}$$

Though the change in units neither removes nor alters any information about physical capital per worker, it does alter the value of the weighted growth in physical capital's share, as evidenced by the above expression, which reveals that

$$\left(\frac{\dot{\alpha}}{\alpha}\right)(\ln k_t)\alpha_t \neq \left(\frac{\dot{\alpha}}{\alpha}\right)[\ln\theta + \ln k_t]\alpha_t .$$

Since $\left(\frac{\dot{\alpha}}{\alpha}\right)(\ln k_t)\alpha_t$ is not robust to a change in the measurement units of *k* and because $\frac{\dot{A}}{A}$ is a residual and thus a function of $\left(\frac{\dot{\alpha}}{\alpha}\right)(\ln k_t)\alpha_t$, it follows that $\frac{\dot{A}}{A}$ is also sensitive to the units used to measure *k*.

In the special case where $\alpha_t = \alpha$ for all *t*, the fourth term on the right side of equation (2) becomes zero. In this case, the common assumption of constant factor shares eliminates the unit-invariance problem. Without this assumption, the standard growth accounting methodology is invalid. A generalized methodology that is insensitive to measurement units for the less restrictive case of non-constant factor shares would be more appealing.

In like manner, the fifth term on the right side of equation (2) is sensitive to the choice of units for measuring natural capital per worker *n* unless $\gamma_t = \gamma$ for all *t*. Though the above explanation of the unit-invariance problem pertains to a three-input scenario, it can be generalized to any number of inputs. In addition, the problem is not specific to a Cobb-Douglas environment, and it can be shown that the growth accounting results stemming from a CES production technology are also sensitive to a change in units if factor shares are allowed to vary.

A Unit-Invariant Framework for Growth Accounting in the Presence of Variable Factor Shares

Irving Fisher (1922), Leo Törnqvist (1936) and Henri Theil (1965) laid the groundwork for making economic comparisons of prices or quantities using index numbers. Paul Samuelson and Subramanian Swamy (1974) referred to the analysis of the relationship between index numbers and their underlying production or utility functions as the economic theory of index numbers. It is the application of this theory that represents the methodological departure of our analysis from the standard in the literature.

Define the relationship between aggregate output and the factors of production in a country at time *t* as

$$Y_t = F\left[A_t, K_t, L_t, N_t, \alpha_t, \beta_t, \gamma_t\right] \tag{4}$$

where Y denotes aggregate output; A is productivity; K, L and N represent physical capital, raw labor and natural capital (land), respectively; and the elasticities of output with respect to K, L and N are given by α , β and γ , respectively. After dividing output and all inputs by L, equation (4) can be re-written in per worker terms as

$$y_t = f\left[A_t, k_t, n_t, \alpha_t, \beta_t, \gamma_t\right] \tag{5}$$

where lower case letters represent per worker values.

Let *f* be the per-worker form of the unrestricted constant returns to scale translog production function as defined by Laurits Christensen, Dale Jorgenson and Lawrence Lau (1971 and 1973). Douglas Caves et al. (1982) show that if *f* takes this form, translog indices of growth can be derived, and the translog index of output per worker growth can be expressed as the sum of the translog index of input per worker growth and the translog index of productivity growth as

$$\underbrace{\ln y_t - \ln y_{t-1.}}_{\substack{\text{translog index of}\\ \text{output per worker}\\ \text{growth}}} = \underbrace{\frac{1}{2}\sum_j \left(s_{j,t} + s_{j,t-1}\right)\left(\ln x_{j,t} - \ln x_{j,t-1}\right)}_{\text{translog index of input per worker growth}} + \underbrace{\left(\ln A_t - \ln A_{t-1.}\right)}_{\substack{\text{translog index of}\\ \text{productivity growth}}} \quad (6)$$

where x_j represents factor of production *j* relative to raw labor and s_j is the corresponding factor share. The indices in equation (6) are approximations of growth rates between periods *t* and *t-1*.

The identification of the second term on the right side of equation (6) as the "translog index of productivity growth" is consistent with index theory but is somewhat misleading in the context of growth accounting. The index is sometimes referred to as the "Total Factor Productivity (TFP) index," and though TFP is common jargon in the growth accounting literature, even this term is imprecise. Values of *A* are not observed, and the translog index of productivity growth is computed as a residual. The residual encompasses more than just growth in productivity or efficiency. Given the data used to proxy for the observable components in equation (6), including output, factors of production and factor shares, the translog index of productivity growth is the component that takes on whatever value is needed for the equation to hold exactly.

For ease of exposition we will refer to the translog index of output per worker growth as *output*, the translog index of input per worker growth as *observables* and the translog index of productivity growth as *residual*. The exact form of *observables* will change as assumptions about factors and factor shares change, but in general, *output* can be decomposed as

$$\textit{output} = \textit{observables} + \textit{residual} \quad (7)$$

Consider the observables. To maintain comparability throughout this paper, suppose subscript *j* in equation (6) runs over *k* and *n*, and assume that all income in the economy accrues to physical capital, raw labor and natural capital so that $\alpha_t + \beta_t + \gamma_t = 1$.[9] Under these assumptions, the observables can be expressed as

$$observables = \frac{1}{2}(\alpha_t + \alpha_{t-1})(\ln k_t - \ln k_{t-1}) + \frac{1}{2}(\gamma_t + \gamma_{t-1})(\ln n_t - \ln_{t-1}) \quad (8)$$

Suppose the data undergo the same units transformation as above; the original values of *k* and *n* are expressed in dollars but are multiplied by the constant $\theta = 1/1{,}000$ to convert the data into thousands of dollars. Observables are now given by

$$\begin{aligned}
&\frac{1}{2}(\alpha_t + \alpha_{t-1})(\ln \theta k_t - \ln \theta k_{t-1}) + \frac{1}{2}(\gamma_t + \gamma_{t-1})(\ln \theta n_t - \ln \theta n_{t-1}) \\
&= \frac{1}{2}(\alpha_t + \alpha_{t-1})(\ln \theta + \ln k_t - \ln \theta - \ln k_{t-1}) + \frac{1}{2}(\gamma_t + \gamma_{t-1})(\ln \theta + \ln n_t - \ln \theta - \ln n_{t-1}) \\
&= \frac{1}{2}(\alpha_t + \alpha_{t-1})(\ln k_t - \ln k_{t-1}) + \frac{1}{2}(\gamma_t + \gamma_{t-1})(\ln n_t - \ln_{t-1})
\end{aligned} \quad (9)$$

Therefore, the observables are robust to a change in measurement units, and this result can be generalized to any number of inputs.

It is straightforward to show that output is robust to a change in units, and since the residual is a function of output and observables, it follows that the residual is also insensitive to a change in units. Thus, the growth accounting results arising from equation (6) satisfy the unit-invariance pre-requisite, even if factor shares are allowed to vary over time.

Another appealing aspect of this specific index-based approach is the allowance for non-neutral differences in productivity over time. Moreover, the only restriction imposed on the structure of production within the growth accounting framework is constant returns to scale.

Data

As is common with many historical analyses of the industrial revolution, data limitations prevent the collection of annual data for all years and all necessary variables over the relevant period. Given our data

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[9] The assumption that the factor elasticities sum to 1 implies that factor elasticities and factor shares are equivalent. This allows α , β and γ to be substituted for s_j in equation (6).

requirements and the data availability, we are able to compile a complete sample consisting of 11 decadal observations and thus 10 per decade growth rates spanning the period 1760-1860 for Great Britain.

Data: Output, Raw Labor and Output per Worker

Aggregate output (*Y*) is measured as real GDP, and estimates of this variable, reported in 1700 British pounds, are generated using estimates of nominal GDP and the GDP deflator obtained from the online database supporting Broadberry et al. (2015). Raw labor (*L*) is measured as the labor force, which, following Crafts (2004a), we estimate as the percentage of the population aged 15-64 using data provided in E.A. (Tony) Wrigley et al. (1997).[10,11] Output per worker (*y*) is computed dividing our estimate of real GDP by our estimate of the labor force.

[10] These data are quinquennial observations reported on the ones and sixes (i.e. 1761, 1766, 1771, etc.), so we assume the populations in 1761, 1771, etc. are reasonable estimations of the populations in 1760, 1770, etc.

[11] Total employment would be preferable to the labor force as a measure of raw labor, but total employment data are unavailable. The labor force encompasses employed and unemployed persons. Unemployed people do not contribute to the production of a good or service and therefore do not represent raw labor inputs into aggregate production. Allen (2009) measures labor using Deane and Cole's (1967) estimates of the occupied population. In general, the occupied population should be a more accurate estimate of total employment. However, Dean and Cole acknowledge that their data include unemployed and retired persons, so it is not clear that the Dean and Cole estimates are preferable to the labor force estimates used herein. To the extent that the "labor" estimate, be it occupied population, labor force, etc., encompasses individuals who are not part of the production process, the estimate of productivity growth stemming from a growth accounting exercise will exhibit some degree of inaccuracy. Given the available data, our labor estimates minimize the inherent degree of inaccuracy as much as any other estimates in the literature do.

Data: Physical Capital and Land

We divide Charles Feinstein's (1988) current price values of net domestic reproducible assets by the GDP deflator (1700=100) reported in Broadberry et al. (2015) to derive our estimates of the real physical capital stock (*K*). These estimates, now in 1700 British pounds and consistent with the real GDP data described above, are divided by our labor force estimates to construct the per worker estimates (*k*). Estimates of land per worker (*n*) are constructed in like manner using Feinstein's current price values of land.

Data: Factor Shares

We use the share of wages, land rents and physical capital in national income reported by Clark (2010), and described above, as our estimates of labor, land and physical capital shares, respectively.

Growth Accounting Results

Using the data described above, we compute growth indices in accordance with equation (6). We have decadal observations, so there are ten years between periods *t* and *t-1*, and the indices are per decade growth rates. Our approach considers two per worker inputs, capital per worker and land per worker, so the *j* subscript in equation (6) always runs over *k* and *n*. The translog productivity index is backed out as a residual.

Table 1 provides the sources of British economic growth for three different iterations of the growth accounting exercise.[12] Each iteration corresponds to a different assumption about factor shares. First, we proceed in the standard way by assuming $\alpha_t = 0.35$ and $\gamma_t = 0.15$ for all *t*. Second, we maintain the assumption of constant shares, but rather than use the standard parameter values, we use the mean values of physical capital's share and land's share from our sample. Hence, the second iteration assumes $\alpha_t = 0.241$ and $\gamma_t = 0.15$ for all *t*. The third iteration

[12] See Table A1 in Appendix 1 for alternative growth accounting results yielded by factor shares computed in accordance with Broadberry et al.'s measure of GDP.

allows factor shares to vary over time in accordance with the data described above.

Relative to the capital per worker growth rates associated with the standard assumption of $\alpha_t = 0.35$ for all *t*, the capital per worker growth rates associated with either the mean value of α or the treatment of α as a non-constant variable are all smaller in absolute value. This is to be expected because the estimates of α, which serve as the weights on physical capital per worker, are smaller than 0.35 for all decades in the sample. The decrease in the capital per worker growth rate is non-trivial and for some decades it is quite substantial. For the periods 1830-1840 and 1840-1850, the growth rates fall by a little more than a percentage point when the assumption that $\alpha_t = 0.35$ is relaxed. For the period 1800-1810, the growth rate of capital per worker falls by about 2.5 percentage points.

The mean of land's share (γ) in our sample is 0.15, the standard parameter value used in the literature. Therefore, inserting the sample mean in place of the standard parameter value does not alter the land per worker growth rates. However, γ is not constant; over the period 1760-1860, γ varies from a maximum of 0.202 in 1770 to a minimum of 0.077 in 1860. Treating γ as a variable rather than a constant parameter changes the rates of growth in land per worker. The changes are most substantial for the decades 1790-1800 and 1840-1850, where the absolute values of the growth rates increase and decrease by 0.93 and 1.03 percentage points, respectively.

The sensitivity of the growth accounting results to the factor share choice manifests itself most noticeably in the growth rates of the productivity residual. For the specification where we allow factor shares to vary over time, growth in the productivity residual is higher in seven out of ten decades relative to the standard specification where $\alpha_t = 0.35$ and $\gamma_t = 0.15$ for all *t*. Thus, productivity growth per decade during the British industrial revolution was generally faster than what the standard approach in the literature would suggest. Moreover, in two of these decades, 1800-1810 and 1840-1850, the growth in the productivity residual actually changes sign (from negative to positive) when intertemporal variation in factor shares is allowed.

The Explanatory Power of Factor Shares

When factor shares are allowed to vary, the variation in input and productivity growth rates across decades is a function not just of varying inputs but also of varying factor shares. Growth in shares and growth in inputs are entangled in equation (6) and in the input per worker growth rates reported in Table 1, so the analysis presented in our section on growth accounting results sheds no light on the specific role of factor share growth in explaining output per worker growth in each decade. It would be useful to separate the growth in factor shares from the growth in corresponding inputs so that factor shares could be included as additional "sources of growth" in Table 1. This is not possible given our translog framework.[13] However, the variance decomposition discussed below allows us to determine the variation in output per worker growth over the period 1760-1860 explained by variation in the growth of each input as well as the variation in each factor share.

[13] With the translog framework, there is no factor share growth term. That said, it is not possible to have such a term with Cobb-Douglas or CES production functions either because of the units-invariance problem that arises with time-varying factor shares.

Table 1

Sources of British Economic Growth, 1760-1860

	Output per Worker Growth	Sources of Growth							
		Physical Capital per Worker			Land per Worker		Productivity Residual		
		$\alpha = 0.35$ for all t	$\alpha = 0.241$ for all t	α varies over time	$\gamma = 0.15$ for all t	γ varies over time	$\alpha = 0.35$ and $\gamma = 0.15$ for all t	$\alpha = 0.241$ and $\gamma = 0.15$ for all t	α and γ vary over time
1760-1770	0.70%	-0.82%	-0.56%	-0.51%	-1.32%	-1.74%	2.83%	2.57%	2.95%
1770-1780	5.14%	1.86%	1.28%	1.13%	-0.79%	-1.02%	4.07%	4.64%	5.03%
1780-1790	2.17%	-2.19%	-1.51%	-1.37%	1.71%	2.11%	2.66%	1.98%	1.43%
1790-1800	10.26%	-0.45%	-0.31%	-0.30%	-4.55%	-5.48%	15.26%	15.10%	16.04%
1800-1810	4.19%	8.30%	5.73%	5.88%	-2.02%	-2.31%	-2.10%	0.47%	0.61%
1810-1820	-2.17%	-2.44%	-1.68%	-1.74%	3.26%	3.31%	-3.00%	-3.74%	-3.74%
1820-1830	6.41%	1.42%	0.98%	1.03%	0.91%	0.78%	4.08%	4.52%	4.60%
1830-1840	9.49%	4.14%	2.86%	3.00%	-0.01%	0.00%	5.35%	6.63%	6.49%
1840-1850	6.63%	4.66%	3.21%	3.35%	3.11%	2.08%	-1.14%	0.32%	1.20%
1850-1860	14.65%	1.33%	0.92%	1.01%	-1.95%	-1.10%	15.26%	15.66%	14.73%

Note: The table shows growth rates per decade. $\alpha = 0.35$ and $\gamma = 0.15$ are standard assumptions in the literature. The mean value of α in the sample is 0.241.

Variance Decomposition Methodology

From equation (7) it follows that the variance of output can be decomposed as

$$var[output] = var[observables] + var[residual] + 2cov[observables, residual] \quad (10)$$

Simple estimates of $\frac{var[observables]}{var[output]}$ and $\frac{var[residual]}{var[output]}$ are useful for determining the explanatory power of observables and the residual, respectively, if the covariance term in equation (10) equals zero. However, our data exhibit a statistical correlation between the residual and observables that ranges from -0.617 to -0.641, depending on the specific assumptions used to construct the observables. Because of this correlation, the aforementioned relative variance estimates will not sum to one and will yield inaccurate estimates of explanatory power. The covariance term cannot be ignored. It encompasses interaction effects that need to be accounted for in some manner when determining the contribution of variability in each the observable and residual components to variability in output.

We follow Scott Baier et al. (2006) to construct useful variance decompositions. Consider first the following:

$$\underbrace{\frac{\left(1-\rho^2_{obs.,\ res.}\right)\text{var}[observables]}{\text{var}[output]}}_{RV_{observables}} + \underbrace{\frac{\left\{sd[residual] + sd[observables]\rho_{obs.,\ res.}\right\}^2}{\text{var}[output]}}_{RV_{residual}} = 1 \quad (11)$$

The statistical correlation between observables and the residual is denoted by $\rho_{obs.,\ res.}$. Standard deviation is denoted by *sd*. With this decomposition the covariance between the residual and observables is not ignored. Rather, all of the correlation between observables and the residual is attributed to the residual. The estimates of the relative variances sum to one, and interpreting each value is straightforward. The first term on the left hand side of equation (11), which we label $RV_{observables}$ to denote the relative variance of observables, is the fraction of variation in output attributable to variation in observables, and the second term, labeled

$RV_{residual}$, is the fraction of variation in output attributable to variation in the residual.[14]

Advancements in technology and efficiency during the industrial revolution likely led to changes in the composition and quantity of factors of production, but allocating all of the correlation between observables and the residual to the residual, as in equation (11), is extreme. It is likely that the causation ran in the other direction as well; changes in the composition and quantity of factors of production could have fostered advancements in technology and efficiency. Theory provides little guidance as to how the covariance term should be allocated between observables and the residual, so we compute a second decomposition that considers the opposite extreme, one where all of the correlation between observables and the residual is attributed to observables. This decomposition is given by

$$\underbrace{\frac{\{sd[observables]+sd[residual]\rho_{obs.,\,res.}\}^2}{\text{var}[output]}}_{RV_{observables}}+\underbrace{\frac{(1-\rho^2_{obs.,\,res.})\text{var}[residual]}{\text{var}[output]}}_{RV_{residual}}=1 \quad (12)$$

Equations (11) and (12) allow us to place upper and lower bounds on the variation in output accruing to variation in observables and variation in the residual.

Decomposing the Variation in Observables

Variation in observables is due solely to variation in factors of production when factor shares are treated as constant parameters. If factor shares are allowed to vary over time, variation in observables encompasses variation in factors of production and variation in factor shares. The values of $RV_{observables}$ in equations (11) and (12) reflect the combined explanatory power of all observable components. How important are the variations in each individual factor and factor share in explaining output variation?

[14] The fraction of the variation in observables allocated to the variation in the residual is determined by the squared correlation $(\rho^2_{obs..,\,res.})$. Observables and the residual are negatively correlated. Squaring the correlation ensures that variation in observables that reflects variation in the residual is added to variation in the residual.

Answering this question, which the current industrial revolution literature makes no attempts to do, requires further decomposition of the variance of observables.

We proceed in accordance with Sturgill (2014). The variance of observables can be decomposed as follows:

$$\mathrm{var}\left[observables\right] = \sum_j \mathrm{var}\left[obs_{j,t}\right] + 2\sum_j \sum_q \mathrm{cov}\left[obs_{j,t}, obs_{q,t}\right] \quad (13)$$

where $obs_{j,t}$ is the observable component pertaining to factor of production *j* and is given by $obs_{j,t} = \frac{1}{2}\left(s_{j,t} + s_{j,t-1}\right)\left(\ln x_{j,t} - \ln x_{j,t-1}\right)$.

The additional breakdown of explanatory power is a one or two step process, depending on whether factor shares are treated as constants or variables, respectively. First, the variation in observables must be broken down into the variation attributable to each of the observable components $\left(obs_{j,t}\right)$. If shares are allowed to vary, the second step is breaking down the variation in each observable component into that accruing to the factor and that accruing to the factor share. With constant shares, there is no second step; each s_j is the same for all *t*, and variation in each x_j is the only thing driving variation in $obs_{j,t}$.

First Step

Uniquely estimating the fractions of variation in observables attributable to variation in each $obs_{j,t}$ requires that some assumption about the covariance terms in equation (13) be made. No theory exists to guide this assumption, but upper and lower bounds for the relative variances can be obtained. The details of the methodology for this first step are provided in section 2 of the Appendix.

Second Step

If shares are allowed to vary, breaking down the variation in observables, and ultimately the variation in output, into that accruing to factors and that accruing to factor shares, requires that the variation attributable to factors and factor shares be extracted from the overall variation in each observable component $(obs_{j,t})$. We define the share portion of $obs_{j,t}$ as $\psi_{j,t} = \frac{1}{2}(s_{j,t} + s_{j,t-1})$ and the factor portion as $\phi_{j,t} = \ln x_{j,t} - \ln x_{j,t-1}$ so that $obs_{j,t}$ can be expressed as $obs_{j,t} = (\psi_{j,t})(\phi_{j,t})$. Let E denote the expectations operator and let $\Delta\psi_{j,t} = \psi_{j,t} - E(\psi_{j,t})$ and $\Delta\phi_{j,t} = \phi_{j,t} - E(\phi_{j,t})$. Following the decomposition for the variance of a product presented by Goodman (1960) and Bohrnstedt and Goldberger (1969), the variance of $obs_{j,t}$ can be written as

$$\begin{aligned}\text{var}[obs_{j,t}] &= E^2(\psi_{j,t})\text{var}[\phi_{j,t}] + E^2(\phi_{j,t})\text{var}[\psi_{j,t}] + E[(\Delta\psi_{j,t})^2(\Delta\phi_{j,t})^2] \\ &+ 2E(\psi_{j,t})E[(\Delta\psi_{j,t})(\Delta\phi_{j,t})^2] + 2E(\phi_{j,t})E[(\Delta\phi_{j,t})(\Delta\psi_{j,t})^2] \\ &+ 2E(\psi_{j,t})E(\phi_{j,t})\text{cov}[\psi_{j,t},\phi_{j,t}] - \text{cov}^2[\psi_{j,t},\phi_{j,t}].\end{aligned} \quad (14)$$

The first and second terms on the right hand side of equation (14) can be thought of as the direct effects of variability in $\phi_{j,t}$ and $\psi_{j,t}$, respectively. The remaining terms encompass the interaction between $\phi_{j,t}$ and $\psi_{j,t}$. To uniquely estimate the fractions of variation in $obs_{j,t}$ accruing to $\psi_{j,t}$ and $\phi_{j,t}$, some assumption about the interaction terms must be made. Again, no theory exists to guide such an assumption, but by considering two extreme decompositions, one in which all interaction is assumed to reflect variability in $\psi_{j,t}$ and the other in which all interaction is assumed to reflect variability in $\phi_{j,t}$, the range of possible relative variance estimates can be obtained. These two extreme decompositions are presented in section 3 of the Appendix.

Variance Decomposition Results

Column 1 of Table 2 provides the variance decomposition estimates computed in accordance with the standard assumption that $\alpha_t = 0.35$ and $\gamma_t = 0.15$ for all *t*. Of the variation in Britain's output over the period 1760-1860, 0.3%-36.2% is attributable to observables and 63.8%-99.8%

is attributable to the productivity residual. Thus, variation in productivity growth explains the lion's share of variation in output. The breakdown of the explanatory power of observables, also reported in column 1, suggests that growth in *k* has slightly more explanatory power than growth in *n* when standard factor share assumptions are made. [15] The midpoint of the ranges of variation in output accruing to growth in *k* and growth in *n* are 13.5% and 7.3%, respectively. These results are identical to those yielded by a Cobb-Douglas function of the form $y_t = A_t k_t^{0.35} n_t^{0.15}$. In general, if shares are constant, the variance of observables yielded by the translog index of input per worker growth simplifies to the variance of log-differences yielded by the observable component of a Cobb-Douglas function. If shares vary over time, this is not true and the Cobb-Douglas results are invalid.

[15] Decomposing the variation in observables in accordance with the section on growth accounting results and section 1 of the Appendix reveals that 60.1-74.1% and 30-39.9% of observable variation accrues to $obs_{k,t}$ and $obs_{n,t}$, respectively. Given the constant share assumption, variation in $obs_{k,t}$ and $obs_{n,t}$ stems solely from variation in $(\ln k_t - \ln k_{t-1})$ and $(\ln n_t - \ln n_{t-1})$, respectively. Therefore, the lower bound for the range of variation in output accruing to growth in *k* is given by the product (0.3%)(60.1%) = 0.2%. The upper bound for the range of variation in output accruing to growth in *k* is given by the product (36.2%)(74.1%) = 26.8%. The range of variation in output accruing to growth in *n* is 0.1-14.4% and is determined in the same fashion.

Table 2

Variance Decomposition of British Economic Growth, 1760-1860

	Specification		
	$\alpha = 0.35$ and $\gamma = 0.15$ (1)	$\alpha = 0.241$ and $\gamma = 0.15$ (2)	α and γ vary over time (3)
$RV_{observables}$	0.003-0.362	0.028-0.241	0.019-0.254
Variation accruing to growth in k	0.002-0.268	0.011-0.131	0.005-0.137
Variation accruing to growth in n	0.001-0.144	0.013-0.146	0.007-0.164
Variation accruing to α			0.000-0.024
Variation accruing to γ			0.000-0.023
$RV_{residual}$	0.638-0.998	0.759-0.972	0.746-0.982

Note: RV denotes 'relative variance'. $RV_{observables}$ represents the variation in *output* attributable to differences in *observables*. $RV_{residual}$ represents the variation in *output* attributable to differences in the *residual*. *Observables* refer to the translog index of input per worker growth. *Residual* is the translog index of productivity growth. *Output* is the translog index of output per worker growth.

In column 2 of Table 2 we maintain the constant share assumption but replace the standard parameter values with the mean values of physical capital's share and land's share from our sample.[16] Thus, γ_t remains equal to 0.15 for all *t*, but α_t now equals 0.241 for all *t*. The range of variation in output explained by variation in observables is now smaller with a smaller midpoint. This reduction in the explanatory power of observables is due to the reduction in the explanatory power of growth in *k*. The midpoint and upper bound of the range of variation in output accruing to growth in *k* fall by 6.4 and 13.7 percentage points, respectively, relative to the same measures yielded by the specification in column 1. Results are sensitive to the choice of the share parameters.

Allowing for intertemporal variation in factor shares yields the results in column 3 of Table 2.[17] Physical capital's share and land's share together

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[16] See Table A2 in Appendix 1 for alternative variance decomposition results yielded by factor shares computed in accordance with Broadberry et al.'s measure of GDP.

[17] With varying shares, $RV_{observables}$ now ranges from 1.9% to 25.4%. Decomposing the variation in observables in accordance with the section on growth accounting results and section 1 of the Appendix reveals that 35.5-54.2% of observable variation accrues to $obs_{k,t}$ and 45.8-64.5% of the variation accrues to $obs_{n,t}$. To see where the upper and lower bounds for the range of variation in output accruing to each factor and factor share come from, consider $obs_{k,t}$. According to the decompositions provided in section 2 of the Appendix, 0-17.7% of the variation in $obs_{k,t}$ accrues to $\psi_{k,t}$, the share portion of $obs_{k,t}$. Therefore, the lower bound for the range of variation in output accruing to α is given by the product (1.9%)(35.5%)(0) = 0%. See section 3 of the Appendix for a discussion of this zero lower bound. The upper bound for the range of variation in output accruing to α is given by the product (25.4%)(54.2%)(17.7%) = 2.4%. The ranges of variation in output accruing to γ, growth in *k* and growth in *n* are determined in a similar manner. As reported in column 3 of Table 2,

explain up to 4.7% of the variation in output. At first glance, this may seem small, but variation in observables (factor and factor share growth) explains no more than 25.4% of the variation in output. Thus factor share variation accounts for as much as 19% of the explanatory power accruing to observables. By construction, factor shares have no explanatory power in standard analyses because factor shares are treated as constant parameters.

Conclusion

The validity of the claims made about the sources of economic growth during the industrial revolution is dependent on two things: the quality of the data and the validity of the growth accounting methodology. This paper challenges the standard growth accounting methodology.

Factor shares are usually treated as constant parameters. We treat them as variables. To accommodate intertemporal variation in factor shares, we use translog indices of output, input and productivity growth to perform the growth accounting analysis. Standard growth accounting exercises based on Cobb-Douglas or CES production functions are plagued by an index number problem if factor shares are time-varying. Our use of translog indices circumvents this problem.

As is common in the literature, we determine the relative importance of observable and residual growth in explaining output growth, but we also decompose the variance of observable growth into factor and factor share components. This is not typical in the existing literature and it allows us to determine the explanatory power of factor shares.

Because each decadal observation for physical capital's share, which we obtain from Clark (2010), is smaller than the standard parameter value of 0.35, and given that factor shares serve as the weights on factor growth rates, allowing for intertemporal variation in physical capital's share reduces the capital per worker growth rate in each decade of our sample

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0-2.3% of the variation in output accrues to γ, 0.5-13.7% accrues to growth in k and 0.7-16.4% accrues to growth in n.

(1760-1860). The growth rate is affected the most in the period 1800-1810, where it declines by 2.5 percentage points. Land per worker growth rates are also impacted by the incorporation of time-varying factor shares, though by a smaller degree.

The growth rate of the productivity residual is affected the most. It increases in seven out of ten decades as a result of incorporating intertemporal factor share variation. Moreover, the growth rate of productivity actually changes sign for two decades (1800-1810 and 1840-1850), moving from negative to positive.

Acknowledging the time variation in factor shares, in addition to altering input and productivity growth rates, reveals a new source of economic growth: the change in factor shares in and of itself. We find that variation in factor shares explains up to 4.7 percent of the variation in output per worker growth over the period 1760-1860, which represents 19 percent of the explanatory power accruing to observables. This implies that factor eliminating technical progress, which manifests itself via factor shares, is relevant for the industrial revolution. Thus, connecting innovations in manufacturing and changes in institutions that occurred during the industrial revolution to the behavior of factor shares could help improve our understanding of economic growth during the period.

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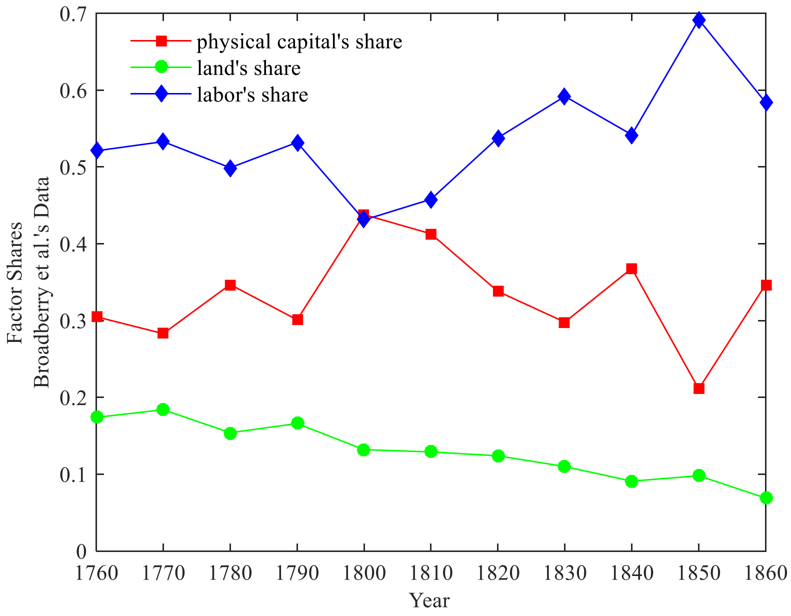
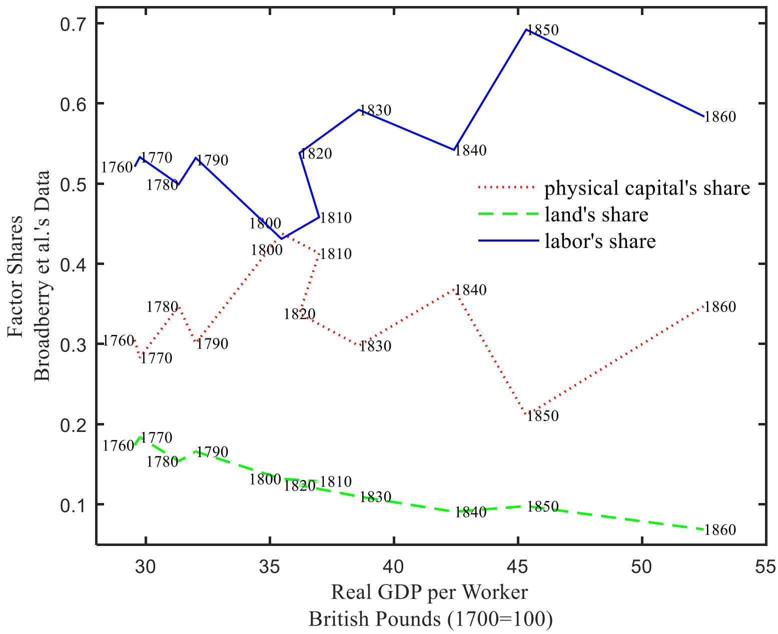
APPENDIX

1. Alternative Results: Factor Shares Computed Using Broadberry et al. GDP Estimates.



Source: Broadberry et al. (2015).

Figure A1
Factor Shares vs. Time (1760-1860)



Source: Broadberry et al. (2015).

Figure A2

Factor Shares vs. Real GDP per Worker

Table A1

Sources of British Economic Growth, 1760-1860: Broadberry et al. Factor Shares

	Output per Worker Growth	Sources of Growth: Physical Capital per Worker			Land per Worker			Productivity Residual		
		$\alpha = 0.35$ for all t	$\alpha = 0.332$ for all t	α varies over time	$\gamma = 0.15$ for all t	$\gamma = 0.13$ for all t	γ varies over time	$\alpha = 0.35$ and $\gamma = 0.15$ for all t	$\alpha = 0.332$ and $\gamma = 0.13$ for all t	α and γ vary over time
1760-1770	0.70%	-0.82%	-0.77%	-0.69%	-1.32%	-1.14%	-1.57%	2.83%	2.61%	2.95%
1770-1780	5.14%	1.86%	1.76%	1.68%	-0.79%	-0.69%	-0.89%	4.07%	4.06%	4.36%
1780-1790	2.17%	-2.19%	-2.08%	-2.03%	1.71%	1.48%	1.83%	2.66%	2.77%	2.38%
1790-1800	10.26%	-0.45%	-0.43%	-0.47%	-4.55%	-3.95%	-4.52%	15.26%	14.63%	15.26%
1800-1810	4.19%	8.30%	7.87%	10.09%	-2.02%	-1.75%	-1.76%	-2.10%	-1.93%	-4.14%
1810-1820	-2.17%	-2.44%	-2.31%	-2.61%	3.26%	2.83%	2.75%	-3.00%	-2.69%	-2.31%
1820-1830	6.41%	1.42%	1.34%	1.29%	0.91%	0.79%	0.71%	4.08%	4.27%	4.41%
1830-1840	9.49%	4.14%	3.93%	3.94%	-0.01%	-0.01%	0.00%	5.35%	5.57%	5.55%
1840-1850	6.63%	4.66%	4.41%	3.85%	3.11%	2.70%	1.96%	-1.14%	-0.48%	0.83%
1850-1860	14.65%	1.33%	1.26%	1.06%	-1.95%	-1.69%	-1.08%	15.26%	15.07%	14.67%

Note: The table shows growth rates per decade. $\alpha = 0.35$ and $\gamma = 0.15$ are standard assumptions in the literature. The mean values of α and γ in the sample are 0.332 and 0.13, respectively.

Table A2

Variance Decomposition of British Economic Growth, 1760-1860: Broadberry et al. Factor Shares

	Specification		
	$\alpha = 0.35$ and $\gamma = 0.15$ (1)	$\alpha = 0.332$ and $\gamma = 0.130$ (2)	α and γ vary over time (3)
$RV_{observables}$	0.003-0.362	0.001-0.336	0.001-0.373
Variation accruing to growth in k	0.002-0.268	0.000-0.262	0.000-0.325
Variation accruing to growth in n	0.001-0.144	0.000-0.118	0.000-0.112
Variation accruing to α			0.000-0.097
Variation accruing to γ			0.000-0.008
$RV_{residual}$	0.638-0.998	0.664-0.999	0.627-0.999

Note: *RV* denotes 'relative variance'. $RV_{observables}$ represents the variation in *output* attributable to differences in *observables*. $RV_{residual}$ represents the variation in *output* attributable to differences in the *residual*. *Observables* refer to the translog index of input per worker growth. *Residual* is the translog index of productivity growth. *Output* is the translog index of output per worker growth.

2. Decomposing the Variation in Observables into the Variation Attributable to each Observable Component $\left(obs_{j,t}\right)$.

Recall from the main text that the variance of observables can be expressed as

$$\text{var}[observables] = \sum_j \text{var}[obs_{j,t}] + 2\sum_j \sum_q \text{cov}[obs_{j,t}, obs_{q,t}].$$

Each observable component is given by

$$obs_{j,t} = \frac{1}{2}\left(s_{j,t} + s_{j,t-1}\right)\left(\ln x_{j,t} - \ln x_{j,t-1}\right)$$

where x_j represents factor of production j relative to raw labor, and s_j is the corresponding factor share.

We consider only two per-worker factors of production, physical capital per worker (*k*) and land per worker (*n*). Denote $\rho_{obs_{k,t}, obs_{n,t}}$ as the statistical correlation between $obs_{k,t}$ and $obs_{n,t}$. If all of the correlation between $obs_{k,t}$ and $obs_{n,t}$ is attributed to $obs_{k,t}$, the relative variances can be computed according to the following decomposition:

$$\frac{\left(1-\rho^2_{obs_{k,t}, obs_{n,t}}\right)\text{var}[obs_{n,t}]}{\text{var}[observables]} + \frac{\left\{sd[obs_{k,t}] + sd[obs_{n,t}]\rho_{obs_{k,t}, obs_{n,t}}\right\}^2}{\text{var}[observables]} = 1 \qquad \text{(A1)}$$

The variation in observables attributable to variation in $obs_{n,t}$ is represented by the first term on the left hand side of equation (A1). The second term represents the variation in observables attributable to variation in $obs_{k,t}$.

Alternatively, all correlation between $obs_{k,t}$ and $obs_{n,t}$ can be attributed to $obs_{n,t}$, in which case the relative variance decomposition takes the form:

$$\frac{\left\{sd[obs_{n,t}] + sd[obs_{k,t}]\rho_{obs_{k,t}, obs_{n,t}}\right\}^2}{\text{var}[observables]} + \frac{\left(1-\rho^2_{obs_{k,t}, obs_{n,t}}\right)\text{var}[obs_{k,t}]}{\text{var}[observables]} = 1 \qquad \text{(A2)}$$

As in equation (A1), the first and second terms in equation (A2) can be interpreted as the fractions of variation in observables attributable to $obs_{n,t}$ and $obs_{k,t}$, respectively.

Once the variance decompositions in equations (A1) and (A2) have been computed, an upper and lower bound for the variation in observables accruing to $obs_{k,t}$ and $obs_{n,t}$ can be determined.

Though we only consider two per-worker factors of production, the general methodology can be applied to a scenario with any number of factors of production. The methodology for three or more factors is slightly different and involves more steps; see Sturgill (2014) for details.

3. Decomposing the Variation in each Observable Component $\left(obs_{j,t}\right)$ into the Variation Attributable to $\psi_{j,t}$ and $\phi_{j,t}$, the Share and Factor Portions of $obs_{j,t}$, Respectively.

In the first decomposition we assume that all interaction between $\psi_{j,t}$ and $\phi_{j,t}$ reflects variability in $\psi_{j,t}$. The relative variance decomposition is given by

$$\frac{E^2(\phi_{j,t})\text{var}[\psi_{j,t}]+Interaction_{\psi_{j,t},\phi_{j,t}}+E^2(\psi_{j,t})\text{var}[\phi_{j,t}]\rho^2_{\psi_{j,t},\phi_{j,t}}}{\text{var}[obs_{j,t}]}+\frac{(1-\rho^2_{\psi_{j,t},\phi_{j,t}})E^2(\psi_{j,t})\text{var}[\phi_{j,t}]}{\text{var}[obs_{j,t}]}=1 \quad \text{(A3)}$$

where

$$\begin{aligned}Interaction_{\psi_{j,t},\phi_{j,t}} &= E\left[(\Delta\psi_{j,t})^2(\Delta\phi_{j,t})^2\right]+2E(\psi_{j,t})E\left[(\Delta\psi_{j,t})(\Delta\phi_{j,t})^2\right]+2E(\phi_{j,t})E\left[(\Delta\phi_{j,t})(\Delta\psi_{j,t})^2\right]\\ &+2E(\psi_{j,t})E(\phi_{j,t})\text{cov}[\psi_{j,t},\phi_{j,t}]-\text{cov}^2[\psi_{j,t},\phi_{j,t}]\end{aligned}$$

and $\rho_{\psi_{j,t},\phi_{j,t}}$ denotes the statistical correlation between $\psi_{j,t}$ and $\phi_{j,t}$. The first term on the left hand side of equation (A3) represents the fraction of variation in $obs_{j,t}$ attributable to variation in $\psi_{j,t}$. The second term represents the fraction of variation attributable to $\phi_{j,t}$.

Alternatively, if all of the interaction is assumed to reflect variability in $\phi_{j,t}$, the relative variances can be estimated according to

$$\frac{(1-\rho^2_{\psi_{j,t},\phi_{j,t}})E^2(\phi_{j,t})\text{var}[\psi_{j,t}]}{\text{var}[obs_{j,t}]}+\frac{E^2(\psi_{j,t})\text{var}[\phi_{j,t}]+Interaction_{\psi_{j,t},\phi_{j,t}}+E^2(\phi_{j,t})\text{var}[\psi_{j,t}]\rho^2_{\psi_{j,t},\phi_{j,t}}}{\text{var}[obs_{j,t}]}=1 \quad \text{(A4)}$$

As in equation (A3), the first term on the left hand side of equation (A4) is the fraction of variation in $obs_{j,t}$ attributable to variation in $\psi_{j,t}$, and the second term is the fraction of variation in $obs_{j,t}$ attributable to variation in $\phi_{j,t}$.

4. The Zero Lower Bound on the Explanatory Power of Factor Shares

The lower bound for the range of variation in output accruing to each factor share is always zero. This result does not imply that factor share variation is unimportant. The lower bound equals zero by construction. Specifically, the zero lower bound stems from three factors: *i)* the reliance of translog indices on differences, *ii)* the structure of the decomposition of the variance of a product of dependent variables, and *iii)* absence of a theory to guide the allocation of the interaction between factor shares and factors. Recall that if all interaction between $\psi_{j,t}$ and $\phi_{j,t}$ is assumed to reflect variability in $\phi_{j,t}$, the relative variance decomposition for the observable component $\left(obs_{j,t}\right)$ is given by equation (A4). The first term on the left hand side of equation (A4) is the fraction of variation in $obs_{j,t}$ attributable to variation in $\psi_{j,t}$, the share portion of $obs_{j,t}$. The numerator of this term is a product that contains $E^2\left(\phi_{j,t}\right)$, which is always equal to zero. Therefore, the lower bound equals zero irrespective of the size of the factor share variance and the strength of the correlation between factor shares and output.

According to the theory of factor saving innovations explored by Zuleta (2008) and Peretto and Seater (2013), factors and factor shares are correlated. The causality runs in both directions. There is a feedback effect, and factors and factor shares drive each other. Thus, neither of the extreme allocations considered in section 3 of the Appendix is likely to be correct. However, there is no theory suggesting a better alternative. Acknowledging that the explanatory power of each factor and factor share falls somewhere within the upper and lower bounds but would almost never equal either bound is likely the most accurate determination.

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